Universidade Federal do ABC
Funções de Uma Variável

3º quadrimestre - 2010

1ª Lista de Exercícios - Limites e Continuidade

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1. Calcule e justifique.

   (a) $\lim_{x\to 2}\ x^2 = 4$

   (b) $\lim_{x\to 1}(3x+1) = 4$

   (c) $\lim_{x\to 10}\ 5 = 5$

   (d) $\lim_{x\to -8}\ \sqrt{5} = \sqrt{5}$

   (e) $\lim_{x\to -1}(-x^2-2x+3) = 4$

   (f) $\lim_{x\to 4}\sqrt{x} = 2$

   (g) $\lim_{x\to 3}\frac{x^2-9}{x+3} = 0$

   (h) $\lim_{x\to \frac{\pi}{4}}\arctan(\log(\tan x)) = 0$

   (i) $\lim_{x\to 3}\frac{x^2-9}{x-3} = 6$

   (j) $\lim_{x\to \frac{1}{2}}\frac{4x^2-1}{2x-1} = 2$

   (k) $\lim_{x\to -\frac{1}{3}}\frac{9x^2-1}{3x+1} = -2$

   (l) $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = \frac{1}{2}$

   (m) $\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3} = \frac{1}{2\sqrt{3}}$

   (n) $\lim_{x\to 3}\frac{\sqrt[3]{x}-\sqrt[3]{3}}{x-3} = \frac{1}{3\sqrt[3]{9}}$

   (o) $\lim_{x\to 2}\frac{\sqrt[4]{x}-\sqrt[4]{2}}{x-2} = \frac{1}{4\sqrt[4]{8}}$

   (p) $\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt{2x+3}-\sqrt{5}} = \frac{\sqrt{5}}{2}$

2. Determine L para que a função dada seja contínua no ponto dado. Justifique.

   (a) $f(x) = \begin{cases} \frac{x^3-8}{x-2} & \text{se } x \neq 2 \\ L & \text{se } x = 2 \end{cases}$ no ponto $p = 2$. $L = 12$

   (b) $f(x) = \begin{cases} \frac{\sqrt{x}-\sqrt{5}}{\sqrt{x+5}-\sqrt{10}} & \text{se } x \neq 5 \\ L & \text{se } x = 5 \end{cases}$ no ponto $p = 5$. $L = \sqrt{2}$

3. $f(x) = \begin{cases} \frac{x^2+x}{x+1} & \text{se } x \neq -1 \\ 2 & \text{se } x = -1 \end{cases}$ é contínua em -1?(não) E em 0?(sim) Por quê?

4. Calcule (em função de x) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ sendo f dada por

   (a) $f(x) = x^2$ $(2x)$

   (b) $f(x) = 2x^2 + x$ $(4x+1)$

   (c) $f(x) = 5$ (0)

   (d) $f(x) = -x^3 + 2x$ $(-3x^2+2)$

   (e) $f(x) = \frac{1}{x}$ $(-\frac{1}{x^2})$

   (f) $f(x) = 3x + 1$ (3)

   Observação: veremos adiante que os limites calculados acima correspondem às *derivadas* das respectivas funções, ou seja à inclinação das retas tangentes aos respectivos gráficos no ponto de abscissa x.

5. Calcule.

   (a) $\lim_{x\to -1} \frac{x^3+1}{x^2-1} = -\frac{3}{2}$

   (b) $\lim_{x\to 0} \frac{x^3+x^2}{3x^3+x^4+x} = 0$

   (c) $\lim_{h\to 0} \frac{(x+h)^3-x^3}{h} = 3x^2$

   (d) $\lim_{x\to 3} \frac{x^2-9}{x^2+9} = 0$

   (e) $\lim_{x\to p} \frac{x^3-p^3}{x-p} = 3p^2$

   (f) $\lim_{x\to p} \frac{\sqrt[3]{x}-\sqrt[3]{p}}{x-p} = \frac{1}{3\sqrt[3]{p^2}}$ para $p \neq 0$.

   (g) $\lim_{h\to 0} \frac{\sqrt[3]{p+h}-\sqrt[3]{p}}{h} = \frac{1}{3\sqrt[3]{p^2}}$ para $p \neq 0$.
   Compare seu resultado com o ítem anterior e veja a observação acima para interpretar geometricamente esses dois limites.

   (h) $\lim_{x\to p} \frac{x^4-p^4}{x-p} = 4p^3$

   (i) $\lim_{x\to p} \frac{\sqrt[4]{x}-\sqrt[4]{p}}{x-p} = \frac{1}{4\sqrt[4]{p^3}}$ para $p \neq 0$.

   (j) $\lim_{h\to 0} \frac{\sqrt[4]{p+h}-\sqrt[4]{p}}{h} = \frac{1}{4\sqrt[4]{p^3}}$ para $p \neq 0$.

   (k) $\lim_{x\to 2} \frac{x^3-5x^2+8x-4}{x^4-5x-6} = 0$

   (l) $\lim_{x\to 1} \frac{x^3-1}{x^4+3x-4} = \frac{3}{7}$

   (m) $\lim_{x\to 7} \frac{\sqrt{x}-\sqrt{7}}{\sqrt{x+7}-\sqrt{14}} = \sqrt{2}$

   (n) $\lim_{x\to p} \frac{x^n-p^n}{x-p} = np^{n-1}$ para $n \in \mathbb{N}$.

(o) $\lim_{x\to p} \frac{\sqrt[n]{x}-\sqrt[n]{p}}{x-p} = \frac{1}{n\sqrt[n]{p^{n-1}}}$ para $p \neq 0$ e $n \in \mathbb{N}$.

(p) $\lim_{x\to 1} \frac{x^m-1}{x^n-1} = \frac{m}{n}$ para $m, n \in \mathbb{N}$.

(q) $\lim_{x\to 0} \frac{x^2}{1-\sqrt{1+x}} = 0$

(r) $\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2} = \frac{1}{2}$

(s) $\lim_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1} = 3$

(t) $\lim_{x\to 5} \frac{\sqrt{x-1}-2}{\sqrt{3x+1}-4} = \frac{2}{3}$

6. Calcule

(a) $\lim_{x\to -1} \sqrt[3]{\frac{x^3+1}{x+1}} = \sqrt[3]{3}$

(b) $\lim_{x\to 1} \frac{\sqrt{x^2+3}-2}{x^2-1} = \frac{1}{4}$

(c) $\lim_{x\to 1} \frac{\sqrt[3]{x+7}-2}{x-1} = \frac{1}{12}$

(d) $\lim_{x\to 1} \frac{\sqrt[3]{3x+5}-2}{x^2-1} = \frac{1}{8}$

(e) $\lim_{x\to 0} \frac{e^{2x}-1}{e^x-1} = 2$

(f) $\lim_{x\to e} \frac{(\log x)^2-1}{\log x-1} = 2$

7. Seja f definida em $\mathbb{R}$. Suponha que $\lim_{x\to 0} \frac{f(x)}{x} = 1$. Calcule

(a) $\lim_{x\to 0} \frac{f(3x)}{x} = 3$

(b) $\lim_{x\to 0} \frac{f(x^2)}{x} = 0$

(c) $\lim_{x\to 1} \frac{f(x^2-1)}{x-1} = 2$

(d) $\lim_{x\to 0} \frac{f(7x)}{3x} = \frac{7}{3}$

8. Utilize o *Teorema do Confronto* e suas generalizações para calcular os seguintes limites:

(a) $\lim_{x\to 0}\ x \cdot \sin\left(\frac{1}{x}\right) = 0$

(b) $\lim_{x\to 1}\ (x-1) \cdot \cos\left(\frac{1}{x-1}\right) = 0$

(c) $\lim_{x\to 0}\ x^2 \cdot \cos\left(\frac{1}{\sqrt[3]{x}}\right) = 0$

(d) $\lim_{x\to +\infty}\ e^{-x} \cdot \cos x = 0$

(e) $\lim_{x\to -\infty}\ e^{x} \cdot \sin\left(x^2+1\right) x = 0$

(f) $\lim_{x\to +\infty}\ e^{\sin x - x} = 0$

(g) $\lim_{x\to +\infty}\ (x + \sin x) = +\infty$

9. A função $f : \mathbb{R} \to \mathbb{R}$ abaixo é limitada, ou seja, existe $M \in \mathbb{R}_+$ tal que $|f(x)| \leq M$ para todo $x \in \mathbb{R}$. Utilize o *Teorema do Confronto* para mostrar que

(a) $\lim_{x\to 0} \; x \cdot f(x) = 0$

(b) $\lim_{x\to 0} \; \sin x \cdot f(x) = 0$

Observe que a função f acima pode ser "altamente patológica", como a função de Dirichlet, definida por $f(x) = \begin{cases} 1, & \text{se } x \text{ for racional} \\ -1, & \text{se } x \text{ for irracional} \end{cases}$

10. Seja $f : \mathbb{R} \to \mathbb{R}$ e suponha que exista $M > 0$ tal que, para todo $x \in \mathbb{R}$, $|f(x) - f(p)| \leq M \cdot |x - p|^2$.

    (a) Mostre que f é contínua em p.

    (b) Calcule, caso exista, $\lim_{x\to p} \frac{f(x)-f(p)}{x-p}$. $= 0$

11. Calcule

    (a) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

    (b) $\lim_{x\to 0} \frac{x}{\sin x} = 1$

    (c) $\lim_{x\to 0} \frac{\sin 3x}{x} = 3$

    (d) $\lim_{x\to \pi} \frac{\sin x}{x-\pi} = -1$

    (e) $\lim_{x\to 0} \frac{x^2}{\sin x} = 0$

    (f) $\lim_{x\to 0} \frac{3x^2}{\tan x \cdot \sin x} = 3$

    (g) $\lim_{x\to 0} \frac{\tan 3x}{\sin 4x} = \frac{3}{4}$

    (h) $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

    (i) $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{2x-\pi} = 0$

    (j) $\lim_{x\to p} \frac{\tan(x-p)}{x^2-p^2} = \frac{1}{2p}$, $\quad p \neq 0$.

    (k) $\lim_{x\to p} \frac{\sin\left(x^2-p^2\right)}{x-p} = 2p$

    (l) $\lim_{x\to 0} \frac{x-\tan x}{x+\tan x} = 0$

    (m) $\lim_{x\to 0} \frac{x+\sin x}{x^2-\sin x} = -2$

    (n) $\lim_{x\to 1} \frac{\sin(\log x)}{\log x} = 1$

    (o) $\lim_{x\to 0} \frac{\sin(\sin x)}{\sin x} = 1$

    (p) $\lim_{x\to 0} \frac{\sin(\sin 2x)}{\sin x} = 2$

    (q) $\lim_{x\to 0} \frac{x}{\arcsin x} = 1$

    (r) $\lim_{x\to 1} \frac{\sin \log \sqrt{x}}{\log x} = \frac{1}{2}$

    (s) $\lim_{x\to 0} \frac{\sin^\circ x}{x} = \frac{\pi}{180}$, onde $\sin^\circ x : \mathbb{R} \to \mathbb{R}$ é a função "seno" que utiliza o grau (e não o radiano) como unidade de medida.

12. Em relação às funções abaixo, faça um esboço do gráfico, indicando os pontos de descontinuidade e os limites indicados, caso existam.

(a) $f(x) = \begin{cases} x^2 & \text{se } x \leq 2 \\ 8-2x & \text{se } 2 < x \end{cases}$

(i) $\lim_{x\to 2^+} f(x) = 4$; (ii) $\lim_{x\to 2^-} f(x) = 4$; (iii) $\lim_{x\to 2} f(x) = 4$.

(b) $f(x) = \begin{cases} x^2-4 & \text{se } x < 2 \\ 4 & \text{se } x = 2 \\ 4-x^2 & \text{se } 2 < x \end{cases}$

(i) $\lim_{x\to 2^+} f(x) = 0$; (ii) $\lim_{x\to 2^-} f(x) = 0$; (iii) $\lim_{x\to 2} f(x) = 0$.
f é descontínua em $x = 2$.

(c) $f(x) = \begin{cases} 2x+3 & \text{se } x < 1 \\ 4 & \text{se } x = 1 \\ x^2+2 & \text{se } 1 < x \end{cases}$

(i) $\lim_{x\to 1^+} f(x) = 3$; (ii) $\lim_{x\to 1^-} f(x) = 5$; (iii) $\lim_{x\to 1} f(x) \nexists$.
f é descontínua em $x = 1$.

(d) $f(x) = \begin{cases} 2 & \text{se } x < -2 \\ \sqrt{4-x^2} & \text{se } -2 \leq x \leq 2 \\ -2 & \text{se } 2 < x \end{cases}$

(i) $\lim_{x\to -2^-} f(x) = 2$; (ii) $\lim_{x\to -2^+} f(x) = 0$; (iii) $\lim_{x\to -2} f(x) \nexists$;
(iv) $\lim_{x\to 2^-} f(x) = 0$; (v) $\lim_{x\to 2^+} f(x) = -2$; (vi) $\lim_{x\to 2} f(x) \nexists$.
f é descontínua em $x = -2$ e em $x = 2$.

(e) $f(x) = \begin{cases} x+1 & \text{se } x < -1 \\ x^2 & \text{se } -1 \leq x \leq 1 \\ 2-x & \text{se } 1 < x \end{cases}$

(i) $\lim_{x\to -1^-} f(x) = 0$; (ii) $\lim_{x\to -1^+} f(x) = 1$; (iii) $\lim_{x\to -1} f(x) \nexists$;
(iv) $\lim_{x\to 1^-} f(x) = 1$; (v) $\lim_{x\to 1^+} f(x) = 1$; (vi) $\lim_{x\to 1} f(x) = 1$.
f é descontínua em $x = -1$.

13. Para $f(x) = \begin{cases} 3x+2 & \text{se } x < 4 \\ 5x+k & \text{se } 4 \leq x \end{cases}$, encontre o valor de k para o qual $\lim_{x\to 4} f(x)$ existe. $k = -6$.

14. Para $f(x) = \begin{cases} x^2 & \text{se } x \leq -2 \\ ax+b & \text{se } -2 < x < 2 \\ 2x-6 & \text{se } 2 \leq x \end{cases}$, encontre os valores de a e b tais que $\lim_{x\to -2} f(x)$ e $\lim_{x\to 2} f(x)$ ambos existam. $a = -3/2$ e $b = 1$.

15. Determine os limites no infinito abaixo.

(a) $\lim_{t\to +\infty} \frac{2t+1}{5t-2} = \frac{2}{5}$

(b) $\lim_{x\to -\infty} \frac{6x-4}{3x+1} = 2$

(c) $\lim_{x\to -\infty} \frac{2x+7}{4-5x} = \frac{-2}{5}$

(d) $\lim_{x\to +\infty} \frac{1+5x}{2-3x} = \frac{-5}{3}$

(e) $\lim_{x\to+\infty} \frac{7x^2-2x+1}{3x^2+8x+5} = \frac{7}{3}$

(f) $\lim_{s\to`-\infty} \frac{4s^2+3}{2s^2-1} = 2$

(g) $\lim_{x\to-\infty} \frac{4x^3+2x^2-5}{8x^3+x+2} = \frac{1}{2}$

(h) $\lim_{x\to+\infty} \frac{3x^4-7x^2+2}{2x^4+1} = \frac{3}{2}$

(i) $\lim_{x\to+\infty} \frac{\sqrt{x^2+4}}{x+4} = 1$

(j) $\lim_{x\to-\infty} \frac{\sqrt{x^2+4}}{x+4} = -1$

(k) $\lim_{w\to-\infty} \frac{\sqrt{w^2-2w+3}}{w+5} = -1$

(l) $\lim_{y\to-\infty} \frac{\sqrt{y^4+1}}{2y^2-3} = \frac{1}{2}$

(m) $\lim_{x\to+\infty} \left(\sqrt{x^2+1}-x\right) = 0$

(n) $\lim_{x\to+\infty} \left(\sqrt{x^2+x}-x\right) = \frac{1}{2}$

(o) $\lim_{x\to+\infty} \left(\sqrt[3]{x^3+1}-x\right) = 0$

(p) $\lim_{x\to-\infty} \left(\sqrt[3]{x^3+x}-\sqrt[3]{x^3+1}\right) = 0$

(q) $\lim_{t\to+\infty} \frac{\sqrt{t+\sqrt{t+\sqrt{t}}}}{\sqrt{t+1}} = 1$

16. Calcule.

(a) $\lim_{x\to+\infty} \left(x^4-3x+2\right) = +\infty$

(b) $\lim_{x\to+\infty} \left(5-4x+x^2-x^5\right) = -\infty$

(c) $\lim_{x\to+\infty} \frac{5x^3-6x+1}{6x^2+x+3} = +\infty$

(d) $\lim_{x\to+\infty} \left(2x-\sqrt{x^2+3}\right) = +\infty$

(e) $\lim_{x\to+\infty} \left(x-\sqrt{3x^3+2}\right) = -\infty$

(f) $\lim_{x\to+\infty} \left(\sqrt{x+\sqrt{x}}-\sqrt{x-1}\right) = \frac{1}{2}$

(g) $\lim_{x\to+\infty} \left(x-\sqrt[3]{3x^3+2}\right) = -\infty$

(h) $\lim_{x\to 0^+} \frac{2x+1}{x} = +\infty$

(i) $\lim_{x\to 0^-} \frac{x-3}{x^2} = -\infty$

(j) $\lim_{x\to 0^+} \frac{3}{x^2-x} = -\infty$

(k) $\lim_{x\to 0^-} \frac{3}{x^2-x} = +\infty$

(l) $\lim_{x\to \frac{1}{2}^+} \frac{3x+1}{4x^2-1} = +\infty$

(m) $\lim_{x\to 1^-} \frac{2x+3}{x^2-1} = -\infty$

(n) $\lim_{x\to 3^+} \frac{x^2-3x}{x^2-6x+9} = +\infty$

(o) $\lim_{x\to -1^+} \frac{2x+1}{x^2+x} = +\infty$

(p) $\lim_{x\to 2^+} \frac{x^2-4}{x^2-4x+4} = +\infty$

(q) $\lim_{x\to -1^+} \frac{3x^2-4}{1-x^2} = -\infty$

(r) $\lim_{x\to 0^+} \frac{\sin x}{x^3-x^2} = -\infty$

(s) $\lim_{x\to \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \cdot \tan x = 1$

(t) $\lim_{x\to +\infty} \left(\frac{\pi}{2} - \arctan x\right) \cdot x = 1$

17. Use o fato de que $\lim_{x\to+\infty} \frac{x}{e^x} = 0$ para calcular os limites abaixo (onde $n \in \mathbb{N}$):

(a) $\lim_{x\to 0^+} (x \cdot \log x) = 0$

(b) $\lim_{x\to +\infty} \left(\frac{x}{\log x}\right) = +\infty$

(c) $\lim_{x\to +\infty} \left(\frac{x}{\log x} - n\right) = +\infty$

(d) $\lim_{x\to +\infty} \log x \cdot \left(\frac{x}{\log x} - n\right) = +\infty$

18. Observe que $\log\left(\frac{e^x}{x^n}\right) = x - n\log x$ e o exercício 17d acima para mostrar que $\lim_{x\to+\infty} \left(\frac{e^x}{x^n}\right) = +\infty$ para todo $n \in \mathbb{N}$.

Observação: Este resultado mostra que a função exponencial cresce mais rapidamente que qualquer polinômio.

19. Calcule $\lim_{x\to+\infty} \frac{x^{100}}{1.01^x} = 0$

Sugestão: Faça $u = (\log 1.01)x$ e utilize o exercício 18 com $n = 100$.

Alerta: Neste caso não confie no que possa sugerir a sua calculadora.

Conclusão: A análise de dados numéricos sem o respaldo teórico adequado pode ser a causa de grandes equívocos.

20. Calcule.

(a) $\lim_{x\to+\infty} \left(1 + \frac{2}{x}\right)^x = e^2$

(b) $\lim_{x\to+\infty} \left(1 - \frac{3}{x}\right)^x = e^{-3}$

(c) $\lim_{x\to+\infty} \left(1 + \frac{1}{x}\right)^{x+2} = e$

(d) $\lim_{x\to+\infty} \left(1 + \frac{1}{2x}\right)^x = \sqrt{e}$

(e) $\lim_{x\to+\infty} \left(1 + \frac{2}{x}\right)^{x+1} = e^2$

(f) $\lim_{x\to+\infty} \left(\frac{x+2}{x+1}\right)^x = e$

(g) $\lim_{x\to+\infty} \left(\frac{x-1}{x+1}\right)^x = e^{-2}$

(h) $\lim_{x\to 0} (1 + 2x)^{\frac{1}{x}} = e^2$

(i) $\lim_{x\to 0} \frac{e^{2x}-1}{x} = 2$

(j) $\lim_{x\to 0} \frac{e^{x^2}-1}{x} = 0$

(k) $\lim_{x\to 0} \frac{5^x-1}{x} = \log 5$

(l) $\lim_{x\to 0^+} \frac{3^x-1}{x^2} = +\infty$

(m) $\lim_{x\to 0^-} \frac{2^x-1}{x^2} = -\infty$

(n) $\lim_{x\to 0^+} \frac{\left(\frac{1}{3}\right)^x-1}{x^2} = -\infty$

(o) $\lim_{x\to 0^-} \frac{\left(\frac{1}{2}\right)^x-1}{x^2} = +\infty$

(p) $\lim_{x\to 0^+} x^x = 1$ **Sugestão:** $x^x = e^{x\cdot\log x}$ e exercício 17a.

21. Utilize o *Teorema do Anulamento* para mostrar que a equação $x^3-4x+2=0$ admite três raízes reais distintas.

22. Seja α a menor raiz positiva da equação $x^3-4x+2=0$. Determine intervalos de amplitude $\frac{1}{2}$, $\frac{1}{4}$ e $\frac{1}{8}$ que contenham α. $\left[\frac{1}{2},1\right]$, $\left[\frac{1}{2},\frac{3}{4}\right]$, $\left[\frac{1}{2},\frac{5}{8}\right]$

23. Mostre que a equação $x^3-\frac{1}{1+x^4}=0$ admite ao menos uma raiz real.

Solução de exercícios selecionados:

1p)

$$\begin{aligned}
\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt{2x+3}-\sqrt{5}} &= \lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt{2x+3}-\sqrt{5}}\cdot\frac{\sqrt{2x+3}+\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}}\\
&= \lim_{x\to 1}\frac{\sqrt{x}-1}{(2x+3)-5}\cdot(\sqrt{2x+3}+\sqrt{5})\\
&= \lim_{x\to 1}\frac{\sqrt{x}-1}{2(x-1)}\cdot\lim_{x\to 1}(\sqrt{2x+3}+\sqrt{5})\\
&= \lim_{x\to 1}\frac{\sqrt{x}-1}{2(\sqrt{x}-1)(\sqrt{x}+1)}\cdot(\sqrt{2\cdot 1+3}+\sqrt{5})\\
&= \lim_{x\to 1}\frac{1}{2(\sqrt{x}+1)}\cdot(2\sqrt{5}) = \frac{1}{2(\sqrt{1}+1)}\cdot(2\sqrt{5}) = \frac{\sqrt{5}}{2} \qquad \square
\end{aligned}$$

4e)

$$\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x-(x+h)}{(x+h)\cdot x}}{h} = \lim_{h\to 0}\frac{-h}{h\cdot x(x+h)}\\
&= \lim_{h\to 0}\frac{-1}{x(x+h)} = -\frac{1}{x(x+0)} = -\frac{1}{x^2} \qquad \square
\end{aligned}$$

5k)

$$\begin{aligned}
\lim_{x\to 2}\frac{x^3-5x^2+8x-4}{x^4-5x-6} &= \lim_{x\to 2}\frac{(x-2)\cdot(x-2)\cdot(x-1)}{(x-2)\cdot(x+1)\cdot(x^2+x+3)}\\
&= \lim_{x\to 2}\frac{(x-2)\cdot(x-1)}{(x+1)\cdot(x^2+x+3)}\\
&= \frac{(2-2)\cdot(2-1)}{(2+1)\cdot(2^2+2+3)} = \frac{0\cdot 1}{3\cdot 9} = 0 \qquad \square
\end{aligned}$$

5p)

$$\begin{aligned}\lim_{x\to 1}\frac{x^m-1}{x^n-1} &= \lim_{x\to 1}\frac{(x-1)\cdot(x^{m-1}+x^{m-2}+\cdots+x^2+x+1)}{(x-1)\cdot(x^{n-1}+x^{n-2}+\cdots+x^2+x+1)}\\ &= \lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+\cdots+x^2+x+1}{x^{n-1}+x^{n-2}+\cdots+x^2+x+1}\\ &= \frac{1^{m-1}+1^{m-2}+\cdots+1^2+1+1}{1^{n-1}+1^{n-2}+\cdots+1^2+1+1}=\frac{m}{n} \qquad \square\end{aligned}$$

5q) Primeira resolução:

$$\begin{aligned}\lim_{x\to 0}\frac{x^2}{1-\sqrt{1+x}} &= \lim_{x\to 0}\frac{x^2}{1-\sqrt{1+x}}\cdot\frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}\\ &= \lim_{x\to 0}\frac{x^2}{1-(1+x)}\cdot(1+\sqrt{1+x})\\ &= \lim_{x\to 0}(-x)\cdot(1+\sqrt{1+x})=(-0)\cdot(1+\sqrt{1+0})=0 \qquad \square\end{aligned}$$

Segunda resolução:

fazendo $\sqrt{1+x}=u \Rightarrow \left\{\begin{array}{l} x=u^2-1\\ \lim_{x\to 0}u=\lim_{x\to 0}\sqrt{1+x}=\sqrt{1+0}=1\end{array}\right\}$ obtemos:

$$\begin{aligned}\lim_{x\to 0}\frac{x^2}{1-\sqrt{1+x}} &= \lim_{u\to 1}\frac{(u^2-1)^2}{(1-u)}\\ &= \lim_{u\to 1}\frac{(u+1)^2\cdot(u-1)^2}{-(u-1)}\\ &= \lim_{u\to 1}-(u+1)^2\cdot(u-1)=-(1+1)^2\cdot(1-1)=-2^2\cdot 0=0 \qquad \square\end{aligned}$$

5t)

$$\begin{aligned}\lim_{x\to 5}\frac{\sqrt{x-1}-2}{\sqrt{3x+1}-4} &= \lim_{x\to 5}\frac{\sqrt{x-1}-2}{\sqrt{3x+1}-4}\cdot\frac{\sqrt{x-1}+2}{\sqrt{3x+1}+4}\cdot\frac{\sqrt{3x+1}+4}{\sqrt{x-1}+2}\\ &= \lim_{x\to 5}\frac{(x-1)-4}{(3x+1)-16}\cdot\frac{\sqrt{3x+1}+4}{\sqrt{x-1}+2}\\ &= \lim_{x\to 5}\frac{(x-5)}{3\cdot(x-5)}\cdot\lim_{x\to 5}\frac{\sqrt{3x+1}+4}{\sqrt{x-1}+2}\\ &= \lim_{x\to 5}\frac{1}{3}\cdot\frac{\sqrt{3\cdot 5+1}+4}{\sqrt{5-1}+2}=\frac{1}{3}\cdot\frac{8}{4}=\frac{2}{3} \qquad \square\end{aligned}$$

6d) fazendo $(3x+5)=u \Rightarrow \left\{\begin{array}{l} x=\frac{(u-5)}{3}\\ \lim_{x\to 1}u=\lim_{x\to 1}(3x+5)=3\cdot 1+5=8\end{array}\right\}$ obtemos:

$$\begin{aligned}\lim_{x\to 1}\frac{\sqrt[3]{3x+5}-2}{x^2-1} &= \lim_{u\to 8}\frac{\sqrt[3]{u}-2}{\left(\frac{u-5}{3}\right)^2-1}\\ &= \lim_{u\to 8}\frac{\sqrt[3]{u}-2}{\frac{u^2-10u+16}{9}}\end{aligned}$$

$$
\begin{aligned}
&= \lim_{u\to 8} \frac{9\cdot(\sqrt[3]{u}-2)}{(u-8)\cdot(u-2)}\\
&= \lim_{u\to 8} \frac{9\cdot(\sqrt[3]{u}-2)}{\left\{(\sqrt[3]{u})^3-2^3\right\}\cdot(u-2)}\\
&= \lim_{u\to 8} \frac{9\cdot(\sqrt[3]{u}-2)}{\left\{(\sqrt[3]{u}-2)\cdot\left[(\sqrt[3]{u})^2+2\sqrt[3]{u}+2^2\right]\right\}\cdot(u-2)}\\
&= \lim_{u\to 8} \frac{9}{\left[(\sqrt[3]{u})^2+2\sqrt[3]{u}+2^2\right]\cdot(u-2)}\\
&= \frac{9}{\left[\left(\sqrt[3]{8}\right)^2+2\sqrt[3]{8}+2^2\right]\cdot(8-2)} = \frac{1}{8} \qquad \square
\end{aligned}
$$

7a) fazendo $(3x = u) \Rightarrow (\lim_{x\to 0} u = \lim_{x\to 0} 3x = 3\cdot 0 = 0)$ obtemos:

$$
\lim_{x\to 0}\frac{f(3x)}{x} = \lim_{x\to 0}\frac{f(3x)}{3x}\cdot 3 = \lim_{u\to 0}\frac{f(u)}{u}\cdot 3 = \lim_{u\to 0}\frac{f(u)}{u}\cdot\lim_{u\to 0} 3 = 1\cdot 3 = 3 \qquad \square
$$

7c) fazendo $\left[\left(x^2-1\right)=u\right] \Rightarrow \left[\lim_{x\to 1} u = \lim_{x\to 1}\left(x^2-1\right) = 1^2-1=0\right]$ obtemos:

$$
\begin{aligned}
\lim_{x\to 1}\frac{f(x^2-1)}{x-1} &= \lim_{x\to 1}\frac{f(x^2-1)}{(x-1)\cdot(x+1)}\cdot(x+1)\\
&= \lim_{x\to 1}\frac{f(x^2-1)}{(x^2-1)}\cdot\lim_{x\to 1}(x+1)\\
&= \lim_{u\to 0}\frac{f(u)}{(u)}\cdot(1+1) = 1\cdot 2 = 2 \qquad \square
\end{aligned}
$$

11i) fazendo $\left[\left(\frac{\pi}{2}-x\right)=u\right] \Rightarrow \left[\lim_{x\to\frac{\pi}{2}} u = \lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right) = \frac{\pi}{2}-\frac{\pi}{2}=0\right]$ obtemos:

$$
\begin{aligned}
\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{2x-\pi} &= \lim_{x\to\frac{\pi}{2}}\frac{2\cdot\sin\left(\frac{\frac{\pi}{2}-x}{2}\right)\cdot\cos\left(\frac{\frac{\pi}{2}+x}{2}\right)}{-2\cdot\left(\frac{\pi}{2}-x\right)}\\
&= \lim_{x\to\frac{\pi}{2}}\frac{2\cdot\sin\left(\frac{\frac{\pi}{2}-x}{2}\right)\cdot\cos\left(\frac{\frac{\pi}{2}+x}{2}\right)}{-4\cdot\left(\frac{\frac{\pi}{2}-x}{2}\right)}\\
&= \lim_{x\to\frac{\pi}{2}} -\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{2}}\frac{\sin\left(\frac{\frac{\pi}{2}-x}{2}\right)}{\frac{\frac{\pi}{2}-x}{2}}\cdot\lim_{x\to\frac{\pi}{2}}\cos\left(\frac{\frac{\pi}{2}+x}{2}\right)\\
&= -\frac{1}{2}\cdot\lim_{u\to 0}\frac{\sin u}{u}\cdot\cos\left(\frac{\frac{\pi}{2}+\frac{\pi}{2}}{2}\right)\\
&= -\frac{1}{2}\cdot 1\cdot\cos\frac{\pi}{2} = 0 \qquad \square
\end{aligned}
$$

11l)

$$
\lim_{x\to 0}\frac{x-\tan x}{x+\tan x} = \lim_{x\to 0}\frac{x-\frac{\sin x}{\cos x}}{x+\frac{\sin x}{\cos x}}
$$

$$
\begin{aligned}
&= \lim_{x\to 0} \frac{x\cdot\left(1-\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)}{x\cdot\left(1+\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)} \\
&= \lim_{x\to 0} \frac{1-\frac{\sin x}{x}\cdot\frac{1}{\cos x}}{1+\frac{\sin x}{x}\cdot\frac{1}{\cos x}} \\
&= \frac{\lim_{x\to 0}\left(1-\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)}{\lim_{x\to 0}\left(1+\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)} \\
&= \frac{\lim_{x\to 0} 1-\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)}{\lim_{x\to 0} 1+\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)} \\
&= \frac{1-\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}}{1+\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}} \\
&= \frac{1-1\cdot\frac{1}{\cos 0}}{1+1\cdot\frac{1}{\cos 0}} = \frac{1-1\cdot\frac{1}{1}}{1+1\cdot\frac{1}{1}} = \frac{1-1}{1+1} = 0 \qquad \square
\end{aligned}
$$

11q) fazendo $(x=\sin u) \Rightarrow \left\{\begin{array}{l} \arcsin x = u \\ \lim_{x\to 0} u = \lim_{x\to 0}\arcsin x = \arcsin 0 = 0 \end{array}\right\}$ obtemos:

$$
\lim_{x\to 0}\frac{x}{\arcsin x} = \lim_{u\to 0}\frac{\sin u}{u} = 1 \qquad \square
$$

11r)

$$
\begin{aligned}
\lim_{x\to 1}\frac{\sin\log\sqrt{x}}{\log x} &= \lim_{x\to 1}\frac{\sin\left(\frac{1}{2}\cdot\log x\right)}{\log x} \\
&= \lim_{x\to 1}\frac{1}{2}\cdot\frac{\sin\left(\frac{1}{2}\cdot\log x\right)}{\frac{1}{2}\cdot\log x} \\
&= \lim_{x\to 1}\frac{1}{2}\cdot\lim_{x\to 1}\frac{\sin\left(\frac{1}{2}\cdot\log x\right)}{\frac{1}{2}\cdot\log x} \\
&= \frac{1}{2}\cdot\lim_{u\to 0}\frac{\sin u}{u} \quad \left(\text{onde } u=\tfrac{1}{2}\log x\right) \\
&= \frac{1}{2}\cdot 1 = \frac{1}{2} \qquad \square
\end{aligned}
$$

11s) Basta observar que $\sin^\circ x = \sin\left(\frac{\pi}{180}\cdot x\right)$ e fazer $\frac{\pi}{180}\cdot x = u$ para obter:

$$
\begin{aligned}
\lim_{x\to 0}\frac{\sin^\circ x}{x} &= \lim_{x\to 0}\frac{\sin\left(\frac{\pi}{180}\cdot x\right)}{x} \\
&= \lim_{x\to 0}\frac{\pi}{180}\cdot\frac{\sin\left(\frac{\pi}{180}\cdot x\right)}{\left(\frac{\pi}{180}\cdot x\right)} \\
&= \lim_{x\to 0}\frac{\pi}{180}\cdot\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{180}\cdot x\right)}{\left(\frac{\pi}{180}\cdot x\right)} \\
&= \frac{\pi}{180}\cdot\lim_{u\to 0}\frac{\sin u}{u} = \frac{\pi}{180}\cdot 1 = \frac{\pi}{180} \qquad \square
\end{aligned}
$$

15h)

$$
\lim_{x\to+\infty}\frac{3x^4-7x^2+2}{2x^4+1} = \lim_{x\to+\infty}\frac{x^4\cdot\left(3-\frac{7}{x^2}+\frac{2}{x^4}\right)}{x^4\cdot\left(2+\frac{1}{x^4}\right)}
$$

$$
\begin{aligned}
&= \lim_{x\to+\infty} \frac{3-\frac{7}{x^2}+\frac{2}{x^4}}{2+\frac{1}{x^4}} \\
&= \frac{\lim_{x\to+\infty}\left(3-\frac{7}{x^2}+\frac{2}{x^4}\right)}{\lim_{x\to+\infty}\left(2+\frac{1}{x^4}\right)} \\
&= \frac{\lim_{x\to+\infty}3-\lim_{x\to+\infty}\frac{7}{x^2}+\lim_{x\to+\infty}\frac{2}{x^4}}{\lim_{x\to+\infty}2+\lim_{x\to+\infty}\frac{1}{x^4}} \\
&= \frac{3-0+0}{2+0} = \frac{3}{2} \qquad \square
\end{aligned}
$$

15n)

$$
\begin{aligned}
\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right) &= \lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right)\cdot\frac{\left(\sqrt{x^2+x}+x\right)}{\left(\sqrt{x^2+x}+x\right)} \\
&= \lim_{x\to+\infty}\frac{\left(x^2+x\right)-x^2}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to+\infty}\frac{x}{x\cdot\left(\sqrt{1+\frac{1}{x}}+1\right)} \\
&= \lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1} \\
&= \frac{1}{\lim_{x\to+\infty}\left(\sqrt{1+\frac{1}{x}}+1\right)} \\
&= \frac{1}{\lim_{x\to+\infty}\left(\sqrt{1+\frac{1}{x}}\right)+\lim_{x\to+\infty}1} \\
&= \frac{1}{\sqrt{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)}+1} \\
&= \frac{1}{\sqrt{\lim_{x\to+\infty}1+\lim_{x\to+\infty}\frac{1}{x}}+1} \\
&= \frac{1}{\sqrt{1+0}+1} = \frac{1}{2} \qquad \square
\end{aligned}
$$

15o)

$$
\begin{aligned}
\lim_{x\to+\infty}\left(\sqrt[3]{x^3+1}-x\right) &= \lim_{x\to+\infty}\left[\left(\sqrt[3]{x^3+1}-\sqrt[3]{x^3}\right)\cdot\frac{\left(\sqrt[3]{x^3+1}\right)^2+\sqrt[3]{x^3+1}\cdot\sqrt[3]{x^3}+\left(\sqrt[3]{x^3}\right)^2}{\left(\sqrt[3]{x^3+1}\right)^2+\sqrt[3]{x^3+1}\cdot\sqrt[3]{x^3}+\left(\sqrt[3]{x^3}\right)^2}\right] \\
&= \lim_{x\to+\infty}\frac{\left(x^3+1\right)-x^3}{\left(x\sqrt[3]{1+\frac{1}{x^3}}\right)^2+x\sqrt[3]{1+\frac{1}{x^3}}\cdot x+(x)^2} \\
&= \lim_{x\to+\infty}\frac{1}{x^2\cdot\left[\left(\sqrt[3]{1+\frac{1}{x^3}}\right)^2+\sqrt[3]{1+\frac{1}{x^3}}+1\right]}
\end{aligned}
$$

$$= \lim_{x\to+\infty}\frac{1}{x^2}\cdot\lim_{x\to+\infty}\frac{1}{\left(\sqrt[3]{1+\frac{1}{x^3}}\right)^2+\sqrt[3]{1+\frac{1}{x^3}}+1}$$

$$= 0\cdot\frac{1}{3}=0 \qquad \square$$

15q)

$$\lim_{t\to+\infty}\frac{\sqrt{t+\sqrt{t+\sqrt{t}}}}{\sqrt{t+1}} = \sqrt{\lim_{t\to+\infty}\frac{t+\sqrt{t+\sqrt{t}}}{t+1}}$$

$$= \sqrt{\lim_{t\to+\infty}\frac{t}{t+1}+\lim_{t\to+\infty}\frac{\sqrt{t+\sqrt{t}}}{t+1}}$$

$$= \sqrt{\lim_{t\to+\infty}\frac{t}{t\cdot\left(1+\frac{1}{t}\right)}+\lim_{t\to+\infty}\frac{t\cdot\sqrt{\frac{1}{t}+\sqrt{\frac{1}{t^3}}}}{t\cdot\left(1+\frac{1}{t}\right)}}$$

$$= \sqrt{\lim_{t\to+\infty}\frac{1}{1+\frac{1}{t}}+\lim_{t\to+\infty}\frac{\sqrt{\frac{1}{t}+\sqrt{\frac{1}{t^3}}}}{1+\frac{1}{t}}}$$

$$= \sqrt{1+0}=1 \qquad \square$$

16c)

$$\lim_{x\to+\infty}\frac{5x^3-6x+1}{6x^2+x+3} = \lim_{x\to+\infty}\frac{x^2\cdot\left(5x-\frac{6}{x}+\frac{1}{x^2}\right)}{x^2\cdot\left(6+\frac{1}{x}+\frac{3}{x^2}\right)}$$

$$= \lim_{x\to+\infty}\frac{5x-\frac{6}{x}+\frac{1}{x^2}}{6+\frac{1}{x}+\frac{3}{x^2}}$$

$$= \frac{\lim_{x\to+\infty}\left(5x-\frac{6}{x}+\frac{1}{x^2}\right)}{\lim_{x\to+\infty}\left(6+\frac{1}{x}+\frac{3}{x^2}\right)}$$

$$= \frac{\lim_{x\to+\infty}5x-\lim_{x\to+\infty}\frac{6}{x}+\lim_{x\to+\infty}\frac{1}{x^2}}{\lim_{x\to+\infty}6+\lim_{x\to+\infty}\frac{1}{x}+\lim_{x\to+\infty}\frac{3}{x^2}}$$

$$= \frac{+\infty-0+0}{6+0+0}=\frac{+\infty}{6}=+\infty \qquad \square$$

16f)

$$\lim_{x\to+\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x-1}\right) = \lim_{x\to+\infty}\left[\left(\sqrt{x+\sqrt{x}}-\sqrt{x-1}\right)\cdot\frac{\left(\sqrt{x+\sqrt{x}}+\sqrt{x-1}\right)}{\left(\sqrt{x+\sqrt{x}}+\sqrt{x-1}\right)}\right]$$

$$= \lim_{x\to+\infty}\frac{(x+\sqrt{x})-(x-1)}{\sqrt{x+\sqrt{x}}+\sqrt{x-1}} = \lim_{x\to+\infty}\frac{\sqrt{x}-1}{\sqrt{x+\sqrt{x}}+\sqrt{x-1}}$$

$$= \lim_{x\to+\infty}\frac{\sqrt{x}\cdot\left(1-\frac{1}{\sqrt{x}}\right)}{\sqrt{x}\cdot\left(\sqrt{1+\frac{1}{\sqrt{x}}}+\sqrt{1-\frac{1}{x}}\right)}$$

$$
\begin{aligned}
&= \lim_{x\to+\infty} \frac{\left(1-\frac{1}{\sqrt{x}}\right)}{\left(\sqrt{1+\frac{1}{\sqrt{x}}}+\sqrt{1-\frac{1}{x}}\right)} \\
&= \frac{\lim_{x\to+\infty}\left(1-\frac{1}{\sqrt{x}}\right)}{\lim_{x\to+\infty}\left(\sqrt{1+\frac{1}{\sqrt{x}}}\right)+\lim_{x\to+\infty}\left(\sqrt{1-\frac{1}{x}}\right)} \\
&= \frac{1}{1+1} = \frac{1}{2} \qquad \square
\end{aligned}
$$

16i)

$$
\lim_{x\to 0^-} \frac{x-3}{x^2} = \lim_{x\to 0^-}(x-3)\cdot \lim_{x\to 0^-}\left(\frac{1}{x^2}\right) = (-3)\cdot(+\infty) = -\infty \qquad \square
$$

16j)

$$
\lim_{x\to 0^+} \frac{3}{x^2-x} = \lim_{x\to 0^+} \frac{3}{x\cdot(x-1)} = \lim_{x\to 0^+}\frac{1}{x}\cdot \lim_{x\to 0^+}\frac{3}{x-1} = (+\infty)\cdot(-3) = -\infty \qquad \square
$$

16p)

$$
\begin{aligned}
\lim_{x\to 2^+} \frac{x^2-4}{x^2-4x+4} &= \lim_{x\to 2^+} \frac{(x-2)\cdot(x+2)}{(x-2)\cdot(x-2)} = \lim_{x\to 2^+}\frac{x+2}{x-2} \\
&= \lim_{x\to 2^+}(x+2)\cdot \lim_{x\to 2^+}\frac{1}{x-2} = 4\cdot(+\infty) = +\infty \qquad \square
\end{aligned}
$$

16s) fazendo $u = \left(\frac{\pi}{2}-x\right) \Rightarrow \left\{ \begin{array}{l} x = \left(\frac{\pi}{2}-u\right) \\ \lim_{x\to\frac{\pi}{2}^-}(u) = \lim_{x\to\frac{\pi}{2}^-}\left(\frac{\pi}{2}-x\right) = 0 \end{array} \right\}$ obtemos:

$$
\begin{aligned}
\lim_{x\to\frac{\pi}{2}^-}\left(\frac{\pi}{2}-x\right)\cdot\tan x &= \lim_{u\to 0} u\cdot\frac{\sin\left(\frac{\pi}{2}-u\right)}{\cos\left(\frac{\pi}{2}-u\right)} \\
&= \lim_{u\to 0} u\cdot\frac{\cos u}{\sin u} = \lim_{u\to 0}\frac{u}{\sin u}\cdot\cos u \\
&= \lim_{u\to 0}\frac{u}{\sin u}\cdot\lim_{u\to 0}\cos u = 1\cdot 1 = 1 \qquad \square
\end{aligned}
$$

17a) fazendo $u = -\log x \Rightarrow \left\{ \begin{array}{l} x = e^{-u} \\ \lim_{x\to 0^+}(u) = \lim_{x\to 0^+}(-\log x) = +\infty \end{array} \right\}$ obtemos:

$$
\lim_{x\to 0^+}(x\cdot\log x) = \lim_{u\to+\infty} e^{-u}\cdot(-u) = -\lim_{u\to+\infty}\frac{u}{e^u} = -0 = 0 \qquad \square
$$

20a) fazendo $u = \frac{x}{2} \Rightarrow \left\{ \begin{array}{l} x = 2u \\ \lim_{x\to+\infty} u = \lim_{x\to+\infty}\frac{x}{2} = +\infty \end{array} \right\}$ obtemos:

$$
\begin{aligned}
\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^x &= \lim_{x\to+\infty}\left(1+\frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}\cdot 2} \\
&= \lim_{u\to+\infty}\left[\left(1+\frac{1}{u}\right)^u\right]^2 = \left[\lim_{u\to+\infty}\left(1+\frac{1}{u}\right)^u\right]^2 = e^2 \qquad \square
\end{aligned}
$$

20b) fazendo $u = \frac{-x}{3} \Rightarrow \left\{ \begin{array}{l} x = -3u \\ \lim_{x\to+\infty} u = \lim_{x\to+\infty} \frac{-x}{3} = -\infty \end{array} \right\}$ obtemos:

$$\begin{aligned}
\lim_{x\to+\infty} \left(1 - \frac{3}{x}\right)^x &= \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{-x}{3}}\right)^{\frac{-x}{3}\cdot(-3)} \\
&= \lim_{u\to-\infty} \left[\left(1 + \frac{1}{u}\right)^u\right]^{-3} = \left[\lim_{u\to-\infty} \left(1 + \frac{1}{u}\right)^u\right]^{-3} = e^{-3} \qquad \square
\end{aligned}$$

20e) fazendo $u = \frac{x}{2} \Rightarrow \left\{ \begin{array}{l} x = 2u \\ \lim_{x\to+\infty} u = \lim_{x\to+\infty} \frac{x}{2} = +\infty \end{array} \right\}$ obtemos:

$$\begin{aligned}
\lim_{x\to+\infty} \left(1 + \frac{2}{x}\right)^{x+1} &= \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}\cdot 2+1} \\
&= \lim_{x\to+\infty} \left[\left(1 + \frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}\cdot 2} \cdot \left(1 + \frac{1}{\frac{x}{2}}\right)\right] \\
&= \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}\cdot 2} \cdot \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{x}{2}}\right) \\
&= \lim_{u\to+\infty} \left[\left(1 + \frac{1}{u}\right)^u\right]^2 \cdot \lim_{u\to+\infty} \left(1 + \frac{1}{u}\right) \\
&= \left[\lim_{u\to+\infty} \left(1 + \frac{1}{u}\right)^u\right]^2 \cdot \lim_{u\to+\infty} \left(1 + \frac{1}{u}\right) \\
&= e^2 \cdot 1 = e^2 \qquad \square
\end{aligned}$$

20g) fazendo $u = \frac{x+1}{-2} \Rightarrow \left\{ \begin{array}{l} x = -2u - 1 \\ \lim_{x\to+\infty} u = \lim_{x\to+\infty} \frac{x+1}{-2} = -\infty \end{array} \right\}$ obtemos:

$$\begin{aligned}
\lim_{x\to+\infty} \left(\frac{x-1}{x+1}\right)^x &= \lim_{x\to+\infty} \left(\frac{x+1-2}{x+1}\right)^x \\
&= \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{x+1}{-2}}\right)^x \\
&= \lim_{u\to-\infty} \left(1 + \frac{1}{u}\right)^{-2u-1} \\
&= \lim_{u\to-\infty} \left\{\left[\left(1 + \frac{1}{u}\right)^u\right]^{-2} \cdot \left(1 + \frac{1}{u}\right)^{-1}\right\} \\
&= \lim_{u\to-\infty} \left[\left(1 + \frac{1}{u}\right)^u\right]^{-2} \cdot \lim_{u\to-\infty} \left(1 + \frac{1}{u}\right)^{-1} \\
&= \left[\lim_{u\to-\infty} \left(1 + \frac{1}{u}\right)^u\right]^{-2} \cdot \left[\lim_{u\to-\infty} \left(1 + \frac{1}{u}\right)\right]^{-1} \\
&= e^{-2} \cdot 1^{-1} = e^{-2} \qquad \square
\end{aligned}$$

20k) fazendo $u = x \log 5 \Rightarrow \{\lim_{x\to 0} u = \lim_{x\to 0} x \log 5 = 0\}$ obtemos:

$$\lim_{x\to 0} \frac{5^x - 1}{x} = \lim_{x\to 0} \frac{e^{x\cdot\log 5} - 1}{x \cdot \log 5} \cdot \log 5$$

$$\begin{aligned} &= \lim_{u\to 0} \frac{e^u - 1}{u} \cdot \log 5 \\ &= 1 \cdot \log 5 = \log 5 \qquad \square \end{aligned}$$

20l)

$$\begin{aligned} \lim_{x\to 0^+} \frac{3^x - 1}{x^2} &= \lim_{x\to 0^+} \left[\frac{3^x - 1}{x} \cdot \frac{1}{x}\right] \\ &= \lim_{x\to 0^+} \frac{3^x - 1}{x} \cdot \lim_{x\to 0^+} \frac{1}{x} \\ &= \log 3 \cdot (+\infty) = +\infty \qquad \square \end{aligned}$$

20m)

$$\begin{aligned} \lim_{x\to 0^-} \frac{2^x - 1}{x^2} &= \lim_{x\to 0^-} \left[\frac{2^x - 1}{x} \cdot \frac{1}{x}\right] \\ &= \lim_{x\to 0^-} \frac{2^x - 1}{x} \cdot \lim_{x\to 0^-} \frac{1}{x} \\ &= \log 2 \cdot (-\infty) = -\infty \qquad \square \end{aligned}$$

20n)

$$\begin{aligned} \lim_{x\to 0^+} \frac{\left(\frac{1}{3}\right)^x - 1}{x^2} &= \lim_{x\to 0^+} \left[\frac{\left(\frac{1}{3}\right)^x - 1}{x} \cdot \frac{1}{x}\right] \\ &= \lim_{x\to 0^+} \frac{\left(\frac{1}{3}\right)^x - 1}{x} \cdot \lim_{x\to 0^+} \frac{1}{x} \\ &= \log\left(\frac{1}{3}\right) \cdot (+\infty) = (-\log 3) \cdot (+\infty) = -\infty \qquad \square \end{aligned}$$

20o)

$$\begin{aligned} \lim_{x\to 0^-} \frac{\left(\frac{1}{2}\right)^x - 1}{x^2} &= \lim_{x\to 0^-} \left[\frac{\left(\frac{1}{2}\right)^x - 1}{x} \cdot \frac{1}{x}\right] \\ &= \lim_{x\to 0^-} \frac{\left(\frac{1}{2}\right)^x - 1}{x} \cdot \lim_{x\to 0^-} \frac{1}{x} \\ &= \log\left(\frac{1}{2}\right) \cdot (-\infty) = (-\log 2) \cdot (-\infty) = +\infty \qquad \square \end{aligned}$$

21) (i) $f(x) = x^3 - 4x + 2 : \mathbb{R} \to \mathbb{R}$ é contínua.

(ii) $[f(-3) < 0, f(-2) > 0] \overset{\text{(TA)}}{\Longrightarrow} [\exists r_1 \in (-3,-2) | f(r_1) = 0]$

(iii) $[f(0) > 0, f(1) < 0] \overset{\text{(TA)}}{\Longrightarrow} [\exists r_2 \in (0,1) | f(r_2) = 0]$

(iv) $[f(1) < 0, f(2) > 0] \overset{\text{(TA)}}{\Longrightarrow} [\exists r_3 \in (1,2) | f(r_3) = 0]$ $\square$